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Quintessence from M-theory

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Abstract: The status of exponential scalar potentials in supergravity is reviewed, and updated. One version of N=8 D=4 supergravity with a positive exponential potential, obtainable from a 'non-compactification' of M-theory, is shown to have an accelerating cosmological solution that realizes 'eternal quintessence'. Some implications for a de Sitter version of the domain wall/QFT correspondence are discussed.

Keywords: M-Theory, AdS-CFT Correspondance, Supergravity Models, Cosmology of Theories beyond the SM.

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Contents

1. Introduction

The current evidence for an accelerating early universe can be accomodated theoretically via a re-introduction of Einstein's (positive) cosmological constant, which is equivalent to the introduction of tensile matter with equation of state $P = -\rho$. More generally, and for general spacetime dimension D, tensile matter with equation of state

$$P = \kappa\rho \tag{1.1}$$

also yields a flat accelerating universe provided that

$$-1 \le \kappa < -\left(\frac{D-3}{D-1}\right). \tag{1.2}$$

Note that

$$R_{00} = \frac{\beta^2}{(D-1)}\left[(D-1)P + (D-3)\rho\right], \tag{1.3}$$

where R_{00} is the time-time component of the Ricci tensor, and

$$\beta = \sqrt{\frac{2(D-1)}{(D-2)}}. \tag{1.4}$$

It follows that acceleration requires negative R_{00} and hence a violation of the strong-energy condition.

One could just set $D = 4$, as other more obvious evidence suggests, but the observations made in this paper are best understood in the context of D-dimensional

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supergravity theories for various D. The equation of state (1.1) can be realized in a model with a real scalar field ϕ and lagrangian density

$$\mathcal{L} = \sqrt{-g}\left\{\frac{1}{2}R - (\partial\phi)^2 - V(\phi)\right\}, \tag{1.5}$$

provided that the scalar potential V takes the form

$$V = \Lambda e^{-2\alpha\phi}, \tag{1.6}$$

for (positive) dimensionless constant α and positive cosmological constant Λ [1]. As will be shown below, the relation between α and κ in D dimensions (in the conventions used here) is

$$\alpha = \beta\sqrt{(1+\kappa)/2}\,. \tag{1.7}$$

Thus $\kappa = -1$ corresponds to $\alpha = 0$, a pure cosmological constant. As we shall see later, the quantity

$$\Delta \equiv \alpha^2 - \beta^2 \tag{1.8}$$

plays an important role in the supergravity context, and the condition (1.2) on κ for acceleration translates to

$$-\beta^2 \le \Delta < -2\,. \tag{1.9}$$

A scalar field with a positive potential that yields an accelerating universe has been called 'quintessence' [2]; the special case under discussion is 'eternal quintessence' because the fixed equation of state implies an eternal expansion, exactly as for the original cosmological constant but with an adjustable acceleration. From an observational standpoint there is no particular merit to eternal quintessence, and other scalar potentials may well be preferable. However, exponential potentials arise naturally in many supergravity models, and have some attractive phenomenological features [3]. Moreover, consistent truncations to a single scalar ϕ of more general potentials are usually sums of exponentials, in which case the potential $V(\phi)$ will approach a pure exponential for large $|\phi|$. A study of the cosmological implications of pure exponential potentials is therefore likely to be relevant in quite general circumstances. One aim of this paper is to examine the implications of supersymmetry for such potentials, in particular maximal supersymmetry, and to find models that realize either eternal quintessence or, in the case of potentials that are only asymptotically exponential, transient quintessence.

As cosmology must ultimately be founded on *quantum* gravity, and as quantum consistency appears to require string/M-theory, we would wish any promising cosmological scenario to be derivable from string/M-theory. In particular, we would wish to be able to embed any D=4 cosmological solution into some solution of D=11 supergravity or IIB D=10 supergravity. However, there is a 'no-go' theorem that, subject to certain premises, rules out the possibility of a positive potential, and hence an accelerating universe, in any effective D=4 supergravity theory obtained in this way [4, 5]. Consider the general case of (warped) compactification from D to $d < D$ spacetime dimensions on some compact non-singular manifold of dimension $n = D - d$, and let R_{MN} and $r_{\mu\nu}$ be the Ricci tensors in D and d dimensions respectively. The no-go theorem then follows from the observation that, for any non-singular D-dimensional metric of the form

$$ds_D^2 = f(y) ds_d^2(x) + ds_n^2(y)\,, \tag{1.10}$$

positivity of R_{00} implies positivity of r_{00}. In other words, for compactifications of the assumed form the strong energy condition in spacetime dimension D implies the strong energy condition in spacetime dimension $d < D$. The latter condition is equivalent to $|g_{00}|V \leq (d-2)\dot{\phi}^2$, and hence $V \leq 0$ if initial conditions can be chosen such that $\dot{\phi} = 0$. More directly, as follows from (1.3) with $D \to d$, the d-dimensional strong energy condition forbids an accelerating d-dimensional universe. As recently emphasized [6, 7], this makes it difficult to embed accelerating cosmologies into string/M-theory because the strong energy condition is satisfied by both D=11 supergravity and IIB D=10 supergravity.

The strong energy condition in D-dimensions may be violated by a $(D-2)$-form gauge potential [4]. D=11 supergravity compactified on any Ricci flat 7-manifold has such a field, and a modified T^7-compactification that exploits this was shown in [8] to yield a 'massive' D=4 N=8 supergravity with a positive exponential potential [8]. However, this model has $\Delta = 0$ and so will not yield an accelerating universe. The strong energy condition may also be violated by scalar field potentials. This possibility is realized in some D=4 N=4 supergravity theories with de Sitter (dS) vacua [9]; these models are truncations of certain non-compact gaugings of N=8 supergravity [10] which are obtainable by 'compactification' of D=11 supergravity [11]. The no-go theorem is evaded in this case [12] because the 'compactifying' space is actually non-compact (see [13] for another type of 'non-compactification' to de Sitter space). As shown in [11], similar 'non-compactifications' yield all the non-compact gauged maximal supergravity theories in D=4,5,7 [14, 15].

Here it will be shown that a particular non-compact gauged N=8 D=4 supergravity has a positive exponential potential satisfying the condition (1.9) needed for an accelerating cosmology. This model provides a realization of eternal quintessence with equation of state $P = -(7/9)\,\rho$; as will be explained later, it is obtainable from a warped 'non-compactification' of M-theory, but it can also be obtained from a similar 'non-compactification' of IIB superstring theory on $H^{(3,3)} \times S^1$, where $H^{(p,q)}$ are the $(p+q-1)$-dimensional non-compact spaces described in [11]. The $H^{(3,3)}$ 'compactification' yields the SO$(3,3)$ gauged D=5 maximal supergravity of [14], and the D=4 model is obtained by a further dimensional reduction on S^1. Moreover, *the accelerating D=4 cosmology is the S^1 reduction of D=5 dS space*; this has some implications for the non-conformal generalization of the dS/CFT correspondence [16, 17] that will be mentioned in the concluding comments.

Of course, as long as the physical acceptability of 'non-compactifications' remains dubious, neither this example of eternal quintessence nor the dS cases discussed in [12] can settle the question of whether string/M-theory is compatible with an accelerating universe. This and other points arising from the results found here will be discussed in a final section For the moment we turn to a study of the implications of quintessence for D-dimensional isotropic and homogeneous cosmologies.

2. D-dimensional quintessence

Consider a D-dimensional FRW spacetime with scale factor $a(t)$ and metric

$$ds^2 = -dt^2 + a(t)^2 d\Omega^2\,, \tag{2.1}$$

where $d\Omega^2$ is the metric of the maximally-symmetric $(D-1)$-space with curvature k^{-1}, for $k=-1,0,1$. If this spacetime is filled with a perfect fluid of mass density ρ and pressure P then the continuity equation for the fluid is

$$\dot\rho = -(D-1)(\rho+P)\left(\frac{\dot a}{a}\right)\,, \tag{2.2}$$

while the Friedmann equation for the scale factor is

$$\left(\frac{\dot a}{a}\right)^2 - \frac{2\rho}{(D-1)(D-2)} = -\frac{k}{a^2}\,. \tag{2.3}$$

These two equations imply that

$$(D-1)(D-2)\ddot a = -a\left[(D-3)\rho + (D-1)P\right]\,, \tag{2.4}$$

and hence an accelerating universe when

$$P < -\frac{(D-3)}{(D-1)}\,\rho\,. \tag{2.5}$$

For the equation of state (1.1) this translates to the condition (1.2) on κ. Also, using (1.1) in (2.2) we deduce that

$$\rho \propto a^{-(D-1)(1+\kappa)}\,. \tag{2.6}$$

Now consider a single scalar model with arbitrary potential V. The scalar equation is (see e.g. [18])

$$\ddot\phi + (D-1)\left(\frac{\dot a}{a}\right)\dot\phi + \frac{\partial V}{\partial\phi} = 0\,, \tag{2.7}$$

while the Friedmann equation is (2.3) with

$$\rho = \dot\phi^2 + V\,. \tag{2.8}$$

Consistency of these equations with the continuity equation (2.2) implies that

$$P = \dot\phi^2 - V\,, \tag{2.9}$$

and hence that

$$\dot\phi^2 = \frac{1}{2}(\rho + P)\,, \qquad V = \frac{1}{2}(\rho - P)\,. \tag{2.10}$$

Given the equation of state (1.1) we then have, in particular, that

$$V = \frac{1}{2}(1-\kappa)\rho\,, \tag{2.11}$$

and hence from (2.6) that

$$V \propto (1-\kappa)a^{-(D-1)(1+\kappa)}\,, \tag{2.12}$$

with a positive constant of proportionality. Similarly, we also have that $\dot\phi^2 = (1+\kappa)\rho/2$ or, equivalently,

$$\rho = \frac{2\dot\phi^2}{1+\kappa}\,. \tag{2.13}$$

Using this in (2.3), and setting $k=0$, we deduce that

$$a^{-(D-1)(1+\kappa)} \propto e^{-\beta\sqrt{2(1+\kappa)}\phi}\,, \tag{2.14}$$

and hence, from (2.12), that[1]

$$V \propto -\Delta\, e^{-2\alpha\phi}\,, \tag{2.15}$$

with a positive constant of proportionality, and with α related to κ by (1.7).

As a check, we look for power-law solutions of the scalar and Friedmann equations of the form

$$a = t^\gamma\,, \qquad e^{\alpha\phi} = t \tag{2.16}$$

for constant γ. The scalar equation (2.7) is solved for V of the form (1.6) if

$$(D-1)\gamma = (1 + \alpha^2\Lambda)\,. \tag{2.17}$$

Using this, the Friedmann equation is solved for $k=0$ if

$$\Lambda = -\alpha^{-4}\Delta\,, \tag{2.18}$$

in agreement with (2.15). In then follows that

$$\gamma = \frac{2}{\alpha^2(D-2)} \tag{2.19}$$

and the condition $\gamma > 1$ for acceleration is thus seen to be equivalent to $\Delta < -2$, as expected. For future purposes we remark that $\gamma = 3$ for $D=4$ and $\alpha = 1/\sqrt{3}$.

[1] Here we use $(1-\kappa) = -2(\alpha^2-\beta^2)/\beta^2$. The $\alpha = \beta$ case is special and must be dealt with separately. One finds that no power-law solutions are possible for any k.

3. Quintessence in $N = 1$ supergravity

Scalar fields of minimal D=4 supergravity models necessarily belong to chiral supermultiplets. As we are interested in a single scalar field we need consider only a single chiral multiplet, for which the physical bosonic fields consist of one scalar ϕ and one pseudoscalar, although to get the general potential for ϕ we will also need to include one vector multiplet. Given that the coupling of supergravity to one chiral supermultiplet and one vector supermultiplet preserves parity,[2] we may consistently set to zero the pseudoscalar in the chiral multiplet. Having done so, we arrive at the bosonic lagrangian

$$\mathcal{L} = \sqrt{-g}\left\{\frac{1}{2}R - \frac{1}{2}h^{-1}(\phi)F^2 - (\partial\phi)^2 - V(\phi)\right\}, \tag{3.1}$$

where the F^2 term is the kinetic term for the vector field of the vector multiplet, which is coupled to the scalar field through the real function $h(\phi)$. The scalar potential V takes the form

$$V = (w')^2 - \beta^2 w^2 + \xi^2 h, \tag{3.2}$$

where $w(\phi)$ is a real superpotential[3] and ξ is a Fayet-Illiopoulos (FI) constant. The superpotential terms are those derived in [20] for general spacetime dimension D on the assumption of positive energy and the existence of an adS vacuum of a certain type. The latter condition can be relaxed, but the existence of the last term, the 'D-term' potential, of (3.2) shows that more general potentials for ϕ are possible, at least in D=4; this is not immediately obvious because a given function h might be equivalent to a modification of the superpotential w, but the examples to be considered below show that this is not so in general.

If we seek a pure exponential potential then we must choose

$$w = me^{-\alpha\phi}, \qquad h = e^{-2\alpha\phi}, \tag{3.3}$$

for mass parameter m. In this case

$$V = \Lambda e^{-2\alpha\phi} \tag{3.4}$$

with

$$\Lambda = m^2(\alpha^2 - \beta^2) + \xi^2\,. \tag{3.5}$$

When $\xi = 0$ we have $\Lambda = \Delta$, so in this case the conditions $\Delta < -2$ and $\Lambda > 0$ that are needed for an accelerating universe are incompatible [6]. But if $\xi \neq 0$ we

[2]If the superpotential is such as to violate parity then the truncation to the single scalar ϕ may not be consistent, in which case the arguments to follow may require modification. See [19] for a discussion of 'two-field' quintessence models.

[3]Initially w is a complex function of the chiral superfield, but after the truncation of the pseodoscalar in the chiral multiplet no generality is lost by restricting it to be a real function.

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can easily arrange for both these conditions to be satisfied [21, 22]. As we shall see later, this possibility is realized by both 'massive' and gauged versions of N=8 D=4 supergravity.

In the absence of the FI term the choice $\alpha = \beta$ is rather special because it leads to a vanishing potential despite a non-vanishing superpotential. This fact allows the following possibility. Consider the superpotential

$$w = m \left[e^{-\beta\phi} - e^{-\alpha\phi} \right] \qquad (\alpha < \beta) \,. \tag{3.6}$$

This yields the potential

$$V = (\alpha + \beta) m^2 \left[2\beta e^{-(\alpha+\beta)\phi} + (\alpha - \beta) e^{-2\alpha\phi} \right] . \tag{3.7}$$

As $\alpha < \beta$, the first term dominates as $\phi \to -\infty$, and we then have an accelerating universe if

$$\alpha < 2\sqrt{\beta^2 - 2} - \beta \,. \tag{3.8}$$

In the $\phi \to \infty$ limit the second term in the potential vanishes and we have

$$V \sim -(\beta^2 - \alpha^2) e^{-2\alpha\phi} \,. \tag{3.9}$$

As V is now negative the universe is decelerating. We thus have a period of acceleration for large negative ϕ, with a transition to deceleration as ϕ passes through zero; the universe then rolls towards $V = 0$ from below.

4. Constraints from extended supersymmetry

As pointed out in [23], the variable $\Delta = \alpha^2 - \beta^2$ has the property that it is unchanged by dimensional reduction, if after the reduction one then consistently truncates to a model with a single scalar field. For this reason it was used in [24] as the basis of a classification of exponential potentials in maximal and half-maximal supergravity theories; the values of Δ identified as occuring in some massive or gauged supergravity fall into the following four classes:

$$\Delta =< -2 \,, \qquad -2 \leq \Delta < 0 \,, \qquad \Delta = 0 \,, \qquad \Delta > 0 \,. \tag{4.1}$$

We shall comment on each of these cases in turn, updating the discussion of [24] where appropriate:

- $\Delta < -2$. This case, which was the focus of [24], is realized by toroidal reductions of the gauged maximal supergravity theories in D=3,4,5 and 7, and also of the gauged 'F(4) supergravity' in D=6. This is because these theories admit adS vacua and hence a truncation to a theory without scalars, or equivalently to a theory with a scalar having a constant negative potential, corresponding to $\alpha = 0$ and $\Lambda < 0$. The values of Δ found this way are all such that $\Delta < -2$.

Of relevance here is the fact, not discussed in [24], that a very similar analysis can be made for those (D=4,5) *non-compact* gauged supergravity theories that admit dS vacua. For exactly the same reasons, these theories can be consistently truncated to a theory without scalars, and a subsequent toroidal reduction and consistent truncation again yields a single scalar model with $\Delta < -2$ but now with a *positive* cosmological constant Λ. We will consider one such case in more detail in the following section.

- $-2 \le \Delta < 0$. This case is realized by several gauged supergravity theories without adS vacua, and probably by other truncations of those with adS vacua. All known examples have $\Delta = -2$, except the maximal gauged D=8 supergravity theory (and hence its toroidal reductions) for which the obvious single scalar truncation yields $\Delta = -4/3$.

- $\Delta = 0$. This was actually the 'fourth type' in the classification of [24]; it was stated there that there was no known supergravity example, but this case *is* realized by the massive N=8 supergravity of [8], which has an exponential potential for a single scalar that can be identified as the volume of the 7-torus in a toroidal compactification from D=11. To verify this we shall apply the construction of [8] to a D-dimensional theory with bosonic lagrangian

$$\mathcal{L}_D = \sqrt{-g}\left\{\frac{1}{2}R - \frac{1}{(p+2)!}F^2_{(p+2)}\right\}, \tag{4.2}$$

where $F_{(p+2)} = dA_{(p+1)}$ is a (p+2)-form field strength for the (p+1)-form gauge potential $A_{(p+1)}$; for $D = 11$ and $p = 2$ this is the bosonic lagrangian density of D=11 supergravity. Let us consider a compactification to (p+2) dimensions, with a reduction/truncation ansatz for which $F_{(p+2)}$ is restricted to the $(p+2)$-dimensional spacetime and the D-metric takes the form

$$ds^2_D = e^{-\frac{4\phi}{\beta p}}dx^\mu dx^\nu g_{\mu\nu} + e^{\frac{4\phi}{\beta(D-p-2)}}\delta_{mn}\,, \tag{4.3}$$

where β is given by (1.4). The lagrangian density governing the dynamics of the $(p+2)$-dimensional fields is

$$\mathcal{L}_d = \sqrt{-g}\left\{\frac{1}{2}R - (\partial\phi)^2 - \frac{1}{(p+2)!}e^{2\beta\phi}F^2_{(p+2)}\right\}. \tag{4.4}$$

The $A_{(p+1)}$ field equation is almost trivial but its general solution introduces a mass parameter m as an integration constant. Taking this into account, an equivalent lagrangian density for the other fields is

$$\mathcal{L}_d = \sqrt{-g}\left\{\frac{1}{2}R - (\partial\phi)^2 - m^2 e^{-2\beta\phi}\right\}. \tag{4.5}$$

Here we have a model with $\alpha = \beta$, and hence $\Delta = 0$.

- $\Delta > 0$. These are the 'massive' supergravity theories with positive potential. The prototype is the massive IIA D=10 theory, for which $\Delta = 4$. Many massive $D \leq 9$ theories with $\Delta = 4/N_c$, for integer N_c, were obtained in [25] by non-trivial dimensional reduction from D=11 ($N_c = 1, 2, 3, 4$ for D=4). All these theories have supersymmetric domain wall 'vacua' in which the only singularity is a delta-function singularity in the curvature at the location of the walls.

5. Eternal quintessence from de Sitter spacetimes

Of all the cases enumerated above only the non-compact gauged supergravity theories with dS vacua, and their dimensional reductions, satisfy the two conditions, $\Lambda > 0$ and $\Delta < -2$, required for an accelerating universe. There may be other possibilities in supergravity theories with fewer supersymmetries; we have seen that the condition for acceleration is not difficult to satisfy within N=1 D=4 supergravity, even for a pure exponential potential, and more general potentials are of course possible. However, it is unclear how these other cases would arise from M-theory, whereas it is known how all maximal supergravity theories are related to M-theory. The non-compact N=8 D=4 supergravity theories with dS vacua have been recently reviewed in the context of the issues discussed here; these provide accelerating cosmologies with equation of state $P = -\rho$. Here we wish to show that another of the non-compact gaugings of N=8 supergravity provides a realization of eternal quintessence.

One way to obtain this model is to start from the D=5 SO(3,3) gauged supergravity, which has a D=5 adS vacuum [14]. Reduction to D=4 yields the $CSO(3,3,2)$ gauged N=8 supergravity (in the notation of [10]). As explained earlier, this model has a consistent truncation to a single scalar field with $\Delta = -8/3$ because this corresponds to $\Delta = -\beta^2$ in D=5, and Δ is unchanged by dimensional reduction. This yields the equation of state

$$P = -(7/9)\,\rho \tag{5.1}$$

and hence acceleration.

Some aspects of this model can be understood as follows. We saw that an accelerating universe can only be obtained from an exponential potential in N=1 supergravity by inclusion of a FI term. In this case, the coefficient α can be identified as the dilaton coupling constant. Now, the only values of the dilaton coupling constant that occur in consistent truncations of N=8 supergravity to a theory with a single scalar are such that $\alpha^2 = 0, \frac{1}{3}, 1, 3$ [26]. Moreover, if we start from a theory in D=5 without scalars and reduce to D=4 then only the values 1/3 and 3 of α^2 occur. The first case corresponds precisely to $\Delta = -8/3$ and hence to $\kappa = -7/9$. The other case corresponds to $\Delta = 0$, but this does not give rise to an accelerating universe. The $\alpha^2 = 1$ case corresponds to $\Delta = -2$ but, again, this does not yield an accelerating universe. Note that if a dS vacuum were possible for $D > 5$ it would yield other values of Δ corresponding to disallowed values of α; this provides another way to see why the non-compact gaugings of $D \geq 6$ supergravity theories do not admit dS vacua.

We will now verify that the accelerating D=4 universe with equation of state (5.1) lifts to D=5 de Sitter space. We start with the $D = 5$ lagrangian density

$$\mathcal{L}_5 = \sqrt{-\det g^{(5)}}\left[R^{(5)} + \Lambda\right]. \tag{5.2}$$

If the 5-metric is written as

$$ds_5^2 = e^{-(2/\sqrt{3})\phi(x)} ds_4^2(x) + e^{(4/\sqrt{3})\phi(x)} dy^2\,, \tag{5.3}$$

where x denotes the 4-space coordinates and y parameterizes a circle, then the 4-metric and scalar ϕ are governed by a $D = 4$ lagrangian density of the form (1.5) with

$$V = \Lambda e^{-(2/\sqrt{3})\phi} \tag{5.4}$$

and hence $\alpha = 1/\sqrt{3}$, as claimed earlier. The accelerating 4-dimensional universe was discussed in section 2 (where it was shown to have $a = t^3$); the 4-metric and dilaton are

$$ds_4^2 = -dt^2 + t^6 ds^2(\mathrm{E}^3)\,, \qquad \phi = \sqrt{3}\log t\,. \tag{5.5}$$

The corresponding 5-metric is

$$ds_5^2 = -(d\log t)^2 + t^4 ds^2(\mathrm{E}^4)\,, \tag{5.6}$$

which is dS_5 space in planar-type coordinates; as such it is obviously a solution of the D=5 theory with lagrangian density (5.2). It will also be a solution of the $\mathrm{SO}(3,3)$ gauged maximal D=5 supergravity, but presumably one at a saddle point of the potential rather than a maximum. This will mean that the dS solution is unstable, but this instability may be a good thing in that it allows an escape from eternal acceleration.

What remains to be explained about the D=4 supergravity model with this accelerating cosmological solution is its relation to M-theory. As mentioned in the introduction, the D=5 $\mathrm{SO}(3,3)$ gauged supergravity can be obtained from a warped 'compactification' of IIB supergravity on $H^{(3,3)}$. However, after the further reduction to $D = 4$ on S^1, we may pass to the dual IIA supergravity (or, more accurately, superstring theory). As this is an S^1 compactification of M-theory we now have M-theory 'compactified' on $H^{(3,3)} \times T^2$, which yields [11] the non-compact gauging of N=8 D=4 supergravity with gauge group $CSO(3,3,2)$; this is the same theory as one obtains by reduction of the $\mathrm{SO}(3,3)$ gauged D=5 supergravity.

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6. Comments

One premise of the no-go theorem of [4, 5] is the time-independence of the compactifying space. This condition is violated by the accelerating D=4 cosmological solution just discussed, so it was *a priori* possible for this solution to arise from a true compactification of D=11 supergravity. The fact that it does not (because of the non-compactness of the internal space) is evidently related to its 5-dimensional interpretation as periodically-identified de Sitter space.

One point that should be appreciated is that the conclusion of the no-go theorem that $V \leq 0$ does not mean that the function $V(\phi)$ must be everywhere non-positive. It is typically the case that V is both unbounded from below *and from above* in compactifications of D=11 supergravity that satisfy all the premises of the theorem. Rather, the theorem states that the *value* of V in any solution of the assumed form is non-positive. Generically, there will be directions in field space for which V is positive and exponentially increasing as one goes to infinity. There are power-law cosmological solutions associated with this limit, with positive V (provided that $\Delta \neq 0$). This does not contradict the no-go theorem because it says nothing about this type of cosmological compactification. Nevertheless, the only *accelerating* D=4 cosmological solution that the author has been able to find in this way is the one described above that is related to M-theory by a 'non-compactification'.

It should also be borne in mind that string/M-theory actually goes beyond supergravity in that it involves branes sources. The equation of state for a gas of p-branes is [27]

$$\kappa = -\frac{p}{(D-1)}, \tag{6.1}$$

which leads to acceleration for $p \geq D-2$. The $p = D-1$ case is a space-filling brane and its tension adds to the cosmological constant, but if the extra dimensions are compact then the sum over the tensions of all branes must vanish. The $p = D-1$ case corresponds to a gas of domain walls for $D = 4$ [28], although it is not clear whether this solves the problem.

Finally, it is instructive to compare the S^1 reduction of dS space to the S^1 reduction of *anti* de Sitter (adS) space considered in [24]; the latter results in a domain wall (DW) solution of the lower dimensional theory. The domain wall metric in the 'dual frame' is again adS but the dilaton resulting from the compactification breaks the adS group to the Poincaré group on the DW; the adS/CFT correspondence is therefore replaced with a DW/QFT correspondence [29]. To apply these ideas to the reduction of dS_5 discussed above, we again introduce a new 'dual-frame' 4-metric

$$d\tilde{s}_4^2 \equiv e^{-(2/\sqrt{3})\phi} ds_4^2 = -d(\log t)^2 + t^4 ds^2(\mathrm{E}^3). \tag{6.2}$$

This is dS_4 but the $\mathrm{SO}(4,1)$ invariance is broken to $ISO(3)$ by the dilaton. Thus, the dS/CFT correspondence [16, 17] is replaced in the non-conformal case by a correspondence between a cosmology and a Euclidean field theory, with the p-brane of the DW/QFT correspondence being replaced by a Cauchy surface of the cosmological spacetime. A change in scale shifts the dilaton and hence moves the Cauchy surface in time.

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